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Leibniz' Principle, General Relativity, and the Observational Dominance of Euclidean Geometry

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Abstract

Leibniz' principle and the observational dominance of Euclidean geometry suggest a huge cosmological constant in Einstein's field equations and a correspondingly huge negative "vacuum" density. This theory lends support to renormalization procedures in quantum electrodynamics and to the view that the interactions we "observe" are fluctuations of the "vacuum state" interpreted as a Fermi sea. Einstein's "preferred" equations, with $\Lambda = 0$, are recovered. "Empty" space has no metric geometry at all.

1. *Introduction*

Leibniz, contemporary of Newton, is credited with a strong belief in the principle that matter determines geometry, a principle dating back to the ancient Greeks. Ernst Mach, in the 1800's, championed a related principle, namely, that distant stars determine the inertia of local bodies. In this paper, the first principle is referred to as the Leibniz principle; the second, as the Mach principle; the two together, as the Leibniz-Mach principles.

The Leibniz-Mach principles are heuristically desirable guides for physical theories, at least as far as this author is concerned. A strong faith in them suggests that if "matter" is to determine geometry, we can learn much about the predominant distribution of matter in the universe by examining the nature of the predominant geometry. In the next section, we pursue this examination and are heuristically led to the idea of a very high density background distribution of "matter." That is, we are led to the idea that the "vacuum" is a uniform high density matter distribution, "high" density meaning a density much greater than that of any currently observed or proposed structures, including neutron stars and even black holes. We find that this suggests a very large value for the cosmological constant in Einstein's field equations. In fact, it suggests that $c^2\Lambda/8\pi\kappa$ is the negative of the large background density, and thus, in magnitude, must be much larger than the largest densities observed or predicted to be observable. Here c is $\sim 3 \times 10^8$ m/sec and κ is Newton's gravitational constant, $\sim\frac{2}{3} \times 10^{-10}$ N m^2/kg^2.

Having proposed this huge Λ on the basis of the Leibniz principle, we proceed in a later section to give a treatment which recovers Einstein's "preferred" equations, the field equations with $\Lambda = 0$. The paper concludes with a discussion indicating future work.

2. *The Dominant Distribution of Matter as Suggested by the Leibniz Principle*

We begin with a strong belief in the Leibniz principle: matter determines geometry. We look at the geometry that dominates all our successful descriptions of natural phenomena: the Euclidean, or Lorentzian, geometry of special relativity. Einstein's principle of equivalence, assumed valid even in the black holes of current geometrodynamic theory (Misner et al., 1973), predicts that the special relativistic Euclidean geometry is always valid locally—it always dominates. In everyday experience here on earth, in our solar system, the Euclidean geometry has yet to fail us significantly even in the large. Thus, the Leibniz principle combined with these observations seems to suggest that nothing in our experience, i.e., no matter distribution in our experience, is very significant compared to the distribution which determines the Euclidean geometry. Thus, one is led to a dominant matter distribution which generates a Euclidean geometry and which is very little affected by even the densest known observable matter distribution.

Consider the contrary. Suppose the Euclidean-generating distribution were very small; i.e., suppose a low density "vacuum," say, e.g., 10^{-30} g/cm^3 (the present-day maximum limit on the intergalactic density). Then one would expect, on the basis of the Leibniz principle, that the appropriate geometry for describing experiments under water, which has a density ~ 1 g/cm^3, would be very different from Euclidean geometry. Experience has failed to prove Euclidean geometry inappropriate under water. In fact, experience has failed to indicate significant deviations from a Euclidean geometric description in the densest distributions yet observed. Even in theories contemplating much denser distributions than those so far observed, e.g., in theories of the black holes of geometrodynamics, Einstein's principle of equivalence is assumed (Misner et al., 1973). That is, it is assumed that the local geometry is still Euclidean. Thus we are experimentally led to the dominance of the Lorentz-Euclidean geometry and hence, via the Leibniz principle, to a huge background matter density, i.e., to a very high density "vacuum."

Let us see what this means in Einstein's field equations. Using the positive time sign convention of Adler et al. (1975), the Einstein equations read

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi\kappa T_{\mu\nu}/c^2 \tag{1}$$

We now consider "flat" space, "flat" here meaning a Euclidean space, with zero curvature. Note well that "flat" does NOT mean completely "empty."

It means Euclidean, and since, in the Leibniz spirit, matter determines geometry, a Euclidean space cannot be "empty." It does have a metric; it does exist. So then some "matter" must exist in a Euclidean space. In fact, Einstein's Eq. (1) relates Λ to the matter distribution which generates a Euclidean geometry. "Euclidean" means $g_{\mu\nu} = \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Lorentz metric:

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

Thus, for a Euclidean space, $G_{\mu\nu} = 0$ and we have the matter distribution $T_{\mu\nu}^{(\mathrm{EUC})}$ given by

$$T_{\mu\nu}^{(\mathrm{EUC})} = -\,(c^2\Lambda/8\pi\kappa)\,\eta_{\mu\nu} \tag{3}$$

As argued above, the Leibniz principle plus the observation of the dominance of Euclidean geometry suggests that $T_{\mu\nu}^{(\mathrm{EUC})}$ must be huge compared to all observed densities. Thus Λ, too, must be huge. The proposal, then, is a very large Λ for the Einstein field Eq. (1), a Λ many orders of magnitude larger than the densest "observable" matter distribution. Such a Λ is consistent with the dominance of Euclidean-Lorentzian geometry in the description of observed physical phenomena. The large $T_{\mu\nu}^{(\mathrm{EUC})}$ is to be taken as the "vacuum"; it is the energy-momentum tensor for "flat" space, for what heretofore has been called "empty" space. The observational matter distributions are to be interpreted as small fluctuations in $T_{\mu\nu}$ away from $T_{\mu\nu}^{(\mathrm{EUC})}$. Section 3 expands on this point.

Note that truly "empty" space, meaning a $T_{\mu\nu} = 0$, has as a solution, since Λ is so large, $g_{\mu\nu} = 0$. That is: NO matter means NO metric, no geometry! This is epistemologically very satisfying, according to the Leibniz principle. I can think of only one other possibility for $g_{\mu\nu}$ in the absence of matter which might be epistemologically preferable, and that is an *undefined* metric. If the field Eq. (1) with huge Λ were not solvable for $g_{\mu\nu}$ when $T_{\mu\nu} = 0$, I would be happier. However, it is clearly solvable with $g_{\mu\nu} = 0$, and this is satisfactory.*

3. *Correspondence with the Einstein "Preferred" Equations*

Einstein evidently preferred the field Eqs. (1) with $\Lambda = 0$, having said, according to Gamow,† that the introduction of the Λ term was "the biggest

* Actually, Eq. (1) is operationally insoluble if $g_{\mu\nu} = 0$, since $g_{\mu\nu} = 0$ means we can perform no length or time measurements, hence can operationally define no coordinate systems, hence can take no derivatives. Thus $G_{\mu\nu}$ is undefined operationally, and we cannot even say that $g_{\mu\nu} = 0$. Operationally, then, Eq. (1) seems to have the desirable property of being indeterminate if $T_{\mu\nu} = 0$; and, after all, operationally is the only way we "really" exist in physics. Of course, if $g_{\mu\nu} = 0$ is not the only solution to Eq. (1) when $T_{\mu\nu} = 0$, these arguments will need revision. A uniqueness proof for (1) when $T_{\mu\nu} = 0$ would thus be highly desirable. Such a proof would assume differentiation to be defined for any $g_{\mu\nu}$, even for $g_{\mu\nu} = 0$.

† G. Gamow quoting Einstein in Misner et al. (1973), pp. 410–411.

blunder of my life." We shall therefore call Eqs. (1) with $\Lambda = 0$ the Einstein preferred equations. In this section we study a treatment of Eq. (1) with large Λ which reproduces the Einstein preferred equations. In this treatment fluctuations in the matter distribution replace the full $T_{\mu\nu}$. We write the field equations in mixed form*:

$$G^{\mu}{}_{\nu} + \Lambda\delta^{\mu}{}_{\nu} = -8\pi T^{\mu}{}_{\nu} \tag{4}$$

Here $\delta^{\mu}{}_{\nu}$ is the Kronecker delta. Also, we write

$$T^{\mu}{}_{\nu} = T^{\mu(\mathrm{EUC})}_{\nu} + f^{\mu}{}_{\nu} \tag{5}$$

where $T^{\mu(\mathrm{EUC})}_{\nu}$ is the large background or "vacuum" part of $T^{\mu}{}_{\nu}$ and $f^{\mu}{}_{\nu}$ is the energy-momentum stress tensor for fluctuations relative to the vacuum. Note that $T^{\mu(\mathrm{EUC})}_{\nu}$ is the energy-momentum stress tensor which generates the Euclidean "flat" geometry. If we now take $T^{\mu(\mathrm{EUC})}_{\nu}$ to be $-\Lambda\delta^{\mu}{}_{\nu}/8\pi$, Eq. (4) reduces to

$$G^{\mu}{}_{\nu} = -8\pi f^{\mu}{}_{\nu} \tag{6}$$

which is precisely Einstein's preferred field equation with fluctuations in the vacuum serving as the observable matter distribution in the universe. Here flat space corresponds to $f^{\mu}{}_{\nu} = 0$, i.e., to the vacuum, and all the results of standard relativity theory are reproduced, with the masses and pressures simply identified now as fluctuations in a very dense background.

Several points need to be made here. First, this procedure has given epistemological support to Einstein's preferred equations. We began by determining Λ from the combination of the observed dominance of Euclidean geometry in our universe plus a desirable epistemological principle, that of Leibniz. This led to a large background, or vacuum, density, which in turn led to interpretation of observed matter as small fluctuations in the very dense background. The full field equations, with large Λ and large $T^{\mu(\mathrm{EUC})}_{\nu}$, then led to Einstein's preferred equations, with no choice any more as to a linear $g_{\mu\nu}$ term.

Second, in this section we have taken Λ to be given by $-8\pi T^{0}{}_{0}{}^{(\mathrm{EUC})}$, whereas previously Λ was taken to be $-8\pi T_{00}{}^{(\mathrm{EUC})}$. In a flat space, with no fluctuations of $T_{\mu\nu}$, these identifications would be equivalent. In a nonflat space they are not, since in general

$$T^{\mu}{}_{\nu} = \sum_{\alpha} g^{\mu\alpha} T_{\alpha\nu} \tag{7}$$

and $g^{\mu\alpha}$ is not in general $\eta^{\mu\alpha}$. We have, in short, three choices, or three theories, to decide among:

$$-8\pi T^{(\mathrm{EUC})}_{\mu\nu} = \Lambda\eta_{\mu\nu} \tag{8}$$

$$-8\pi T^{\mu}{}_{\nu}{}^{(\mathrm{EUC})} = \Lambda\delta^{\mu}{}_{\nu} \tag{9}$$

or

$$-8\pi T^{\mu\nu(\mathrm{EUC})} = \Lambda\eta^{\mu\nu} \tag{10}$$

* Henceforth we use geometrodynamic units, $\kappa = c = 1$. See Misner et al. (1973).

Theory (9), as we have seen, reproduces the standard general relativistic theory. As is shown in a following paper, theory (8) and perhaps theory (10) seem to imply quantum wave mechanics. To make the choice, one guesses we will need to appeal to experiment. Consideration of conservation laws may suffice.

Third, note that if Λ is taken as a large positive number then the vacuum has a large negative mass density. If Λ is taken as large and negative, we have a large positive background density. The Fermi sea, the vacuum state of Dirac electron theory, is a large, in fact infinite, negative mass-energy distribution. A positive sign for Λ would be consistent with, in fact would lend support to, this Fermi sea theory. Further, the huge but finite Λ proposed here suggests a noninfinite Fermi sea, and thus suggests the associated possibilities of a finite renormalization procedure and a finite justification for perturbation theory in quantum physics. Also, as we shall see in a following paper, a positive Λ seems to imply quantum wave equations. Thus, one is inclined to pick the positive sign for Λ and thus make the vacuum a finite high density negative mass-energy background, i.e., a Fermi sea.

4. *Discussion*

This paper gives a satisfying epistemological basis for a high density vacuum and for Einstein's preferred field equations. The high density vacuum of negative sign, associated with a huge positive Λ, is interpretable as a finite Fermi sea. The finiteness of this Fermi sea lends support to renormalization theory in quantum electrodynamics, as it suggests that one can renormalize with huge, but finite, and not infinite, terms. The finiteness of the vacuum density also holds hope of a rigorous justification of perturbation theory in quantum electrodynamics.

Furthermore, the Leibniz principle plus general relativity, combining to suggest the large Λ concept, hold the promise of eliminating the need to consider cosmological boundary conditions; i.e., they suggest that the nature of the "distant" universe is intimately connected with local physics. One conjectured effect of distant parts of the universe, i.e., of cosmological boundary conditions, on local physics is their determination of inertial mass. This is Mach's principle. Thus, the huge Λ theory proposed here may incorporate Mach's principle at a fundamental epistemological level.

In closing, let us briefly consider some possibilities for further work. A strong candidate for the development of the large Λ field equations is an approach from the quaternion points of view recently developed by Sachs (1967–72) and Edmonds (1974). In fact, a following paper (Nickerson, 1975b) outlines a somewhat more general approach utilizing complex quaternion algebra, which is the highest dimensional division algebra possible (Paige & Swift, 1961). The program, following Sachs, is to factor the large Λ field equations into linear quaternion, i.e. spinor, factors and then to use nonlinear couplings to achieve derived masses and, hopefully, to derive the fundamental constants of physics. Actually, as indicated in another paper (Nickerson, 1975a), hopefully, quantum mechanics will be incorporated in this program at an early stage.

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